ERCIM fellowship Programme
Final scientific report

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Host Organisation	Department of Mathematical Sciences, NTNU

Scientific coordinator	Kristian **Seip**

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

The research carried out during the fellow's stay at NTNU can be summarized in the following projects:

- In 2017, Brevig, Ortega-Cerdà, Seip and Zhao conjectured that the inequality

$$||f||_{H^p} \leq \sqrt{\sum_{n \geq 0} |a_n|^2 c_{p/2}(n)},$$

where $c_\alpha(n) := \binom{n + \alpha - 1}{n}$, holds for every holomorphic function in the open unit disk if $p \geq 2$. The fellow has shown that for any such function we have that

$$||f||_{H^p} \leq \sqrt{\sum_{n \geq 0} \frac{|a_n|^2}{c_{2/p}(n)}}$$

when $p \geq 2$. In particular, this implies that the above conjecture is true. The manuscript containing this result and its most immediate consequences has been accepted for publication in Rev. Mat. Iberoam.

- Let P_+ denote the Riesz projection, that is, the operator (densely) defined as

$$P_+ F (e^{it}) := \sum_{n \geq 0} \hat{F}(n) \, e^{int}$$

for $F \in L^1(T)$. It is a well-known fact that P_+ is a bounded operator from $L^p(T)$ to H^p for all finite $p > 1$, and its norm is equal to $csc(\pi/p)$. If X is a is a space of functions containing H^p, the computation of the norm of P_+ as an operator from $L^p(T)$ to X is still an open problem. As a part of an ongoing project jointly with Kristian Seip and Ole Fredrik Brevig (University of Oslo), the fellow is studying the $L^p(T)$ to H^q estimates for P_+.

- In 1984, Sundberg characterized the values that a $BMO(T)$ function can take on an interpolating sequence. Together with Alberto Dayan (Saarland University and former ERCIM fellow at NTNU) and Karl-Mikael Perfekt (NTNU), an analogous result has been deduced for $BMO(D)$ functions that are quasi-Lipschitz with respect to the hyperbolic distance of the disk. Specifically, it is shown that if $\{z_n\}_{n \geq 0}$ is an interpolating sequence for the Bergman space A^2, R_n is the top half of the Carleson box centred at z_n and $\{w_n\}_{n \geq 0}$ is a sequence of complex numbers, then there exists a quasi-Lipschitz and $BMO(D)$ function f whose average on R_n coincides with w_n for all n if and only if there exist $\lambda > 0$ and a function β such that the quantities

$$\sum_{n \geq 0} exp(\lambda|\beta(z) - w_n|) \, (1 - \rho(z, z_n))^2, \quad |z| < 1,$$

where ρ denotes the pseudo-hyperbolic distance of the disk, are uniformly bounded. Regarding another spaces of functions, a similar condition for the Bloch space is the object of the current investigation.

- If $f(z) = \sum_{n\geq 0} a_n z^n$ and $g(z) = \sum_{n\geq 0} b_n z^n$ are two convex mappings, then their convolution (also known as Hadamard product)

$$f * g(z) = \sum_{n\geq 0} a_n b_n z^n$$

  is also convex. This was originally conjectured by Pólya and Schoenberg and was finally proved by Ruscheweyh and Sheil-Small. Jointly with Martin Chuaqui (Pontificia Universidad Católica de Chile), Rodrigo Hernández (Universidad Adolfo Ibáñez) and Alejandro Mas (Universidad de Málaga), the fellow is investigating further properties of the convolution of convex mappings.

- In 1980, Weissler showed that the dilation operator $D_r f(z) := f(rz)$ is contractive from H^p to H^q, $0 < p < q$, when $r \leq \sqrt{p/q}$. This kind of estimates are also known in the literature as hypercontractive inequalities for Hardy spaces. Later, Weissler's inequality was partially extended to standard weighted Bergman spaces by Bayart, Brevig, Haimi, Ortega-Cerdà and Perfekt (2019) and Melentijević (2022). The main goal of this project consists of the study of hypercontractive inequalities for general spaces of analytic functions. It is a work in progress jointly with Alejandro Mas and Petar Melentijević (University of Belgrade).

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

The following paper has been accepted:
- A. Llinares: *Contractive inequalities between Dirichlet and Hardy spaces,* Rev. Mat. Iberoam. (2023), DOI 10.4171/RMI/1418, in press.
  Abstract: We prove a conjecture of Brevig, Ortega-Cerdà, Seip and Zhao about contractive inequalities between Dirichlet and Hardy spaces and discuss its consequent connection with the Riesz projection.

Moreover, it is expected that the results of some of the projects described in the previous section will be submitted to a journal for review over the next few months.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

The fellow has participated in the following conferences and workshops:

- **June 2022:** *Excursions in Analysis*, Trondheim (Norway).
- **September 2022:** *Complex Analysis Workshop*, Viña del Mar (Chile).
  Talk: *Normas de inclusión entre espacios de funciones analíticas* [Norms of inclusions between spaces of analytic functions].
- **December 2022:** *Analysis and Number Theory*, Oslo (Norway).
  Talk: *Contractive inequalities between Dirichlet and Hardy spaces.*

The fellow has attended regularly the Analysis Seminar of the Department of Mathematical Sciences at NTNU. Furthermore, he has given the following seminar talks:

- **November 2022:** *Analysis Seminar*, NTNU (Norway).
  Title: *Contractive inequalities between Dirichlet and Hardy spaces.*
- **December 2022:** *Analysis Seminar,* University of Belgrade (Serbia).
  Online talk: *Contractive inequalities between Dirichlet and Hardy spaces.*
- **March 2023:** *Geometric and Functional Analysis Seminar*, University of Helsinki (Finland).
  Talk: *Contractive inequalities for analytic functions.*

IV – RESEARCH EXCHANGE PROGRAMME (REP)

The fellow visited VTT, located in Helsinki (Finland), from February 13 to 17. The stay was hosted by Jorma Kilpi. During this visit, the fellow gave an introductory talk about his research (titled *"Contractive inequalities in spaces of analytic functions with applications"*). Overall, this stay was a good opportunity to see how Mathematics and Statistics can be applied to bring solutions to challenging problems for business and society.